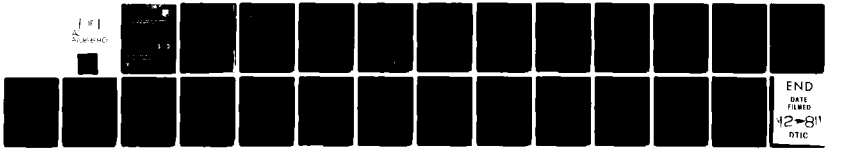
AD-A106 690 ROME AIR DEVELOPMENT CENTER GRIFFISS AFB NY F/G 17/7
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RADC-TR-81-225
In-House Report
August 1981



PASSIVE TRILATERATION POSITIONING PROCEDURE ERROR ANALYSIS

Dean C. Merchant, Colonel, USAFR

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RADC-TR-81-225	2. GOVT ACCESSION NO. AD-A106 690	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) PASSIVE TRILATERATION POSITIONING PROCEDURE ERROR ANALYSIS		5. TYPE OF REPORT & PERIOD COVERED In-House Report
		6. PERFORMING ORG. REPORT NUMBER N/A
7. AUTHOR(s) Dean C. Merchant, Colonel, USAFR		8. CONTRACT OR GRANT NUMBER(s) N/A
9. PERFORMING ORGANIZATION NAME AND ADDRESS Rome Air Development Center (IRRA) Griffiss AFB NY 13441		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62702F 45947TK
11. CONTROLLING OFFICE NAME AND ADDRESS Rome Air Development Center (IRRA) Griffiss AFB NY 13441		12. REPORT DATE August 1981 13. NUMBER OF PAGES 19
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Same (9) Technical Rept's		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Same

18. SUPPLEMENTARY NOTES

Prepared as part of feasibility study of portable positioning test and evaluation system using laser distance ranging to target airborne targets and theoretical accuracy figures attainable. Applies to all DME systems.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Positioning, trilateration, DME, Error-Analysis, Geodetic, error propagation, passive tracking.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents the results of a conceptual and analytical study pertaining to the propagation of errors through a spatial trilateration network. The network consists of an aircraft, DME ground based units and an independent ground based tracker. The primary error sources treated are DME range errors, geodetic coordinate errors of the ground based DME units, timing errors and the "Geodetic slice" between the local survey relating DME stations and the ground based tracker. The result is an

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algebraic tool for studying the influences of various system errors and geometry on the error in positioning an aircraft with respect to a ground based tracking unit.

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I. INTRODUCTION

A need exists at RADC to position an aircraft with respect to a ground based tracker. Such information can be used for adjusting and subsequently calibrating and evaluating the tracking performance of the system. A variety of operational aircraft will be used which sharply limit the equipment that may be conveniently carried by the aircraft. Accordingly, a passive DME approach is planned, using ground based, laser equipped units, reflecting off retroreflective tape affixed to the skin of the aircraft.

The objective of this study is to provide an analytical tool to assess the aircraft positional error with respect to the tracker position due to several primary error sources. This report discusses the concepts, theory and mathematical formulations which lead to this objective. Subsequent work will include programming of these results for graphical presentations in which the factors of error magnitudes and network geometry can be exercised.

II. TECHNICAL DISCUSSION

The description of the trilateration network is made with reference to Figure II-1. A minimum of three ground based DME units (i) are to be used to passively track a single aircraft (a/c). The objective of this study has been to provide the conceptual and procedural framework for assessing the influences of the various error sources on the coordinates of location of the a/c with respect to the tracker at any time (t).

The definitions of symbols and terms may be found in Appendix B.

A. ERROR SOURCES

The primary errors are in the observations of range by the DME units, the internal survey coordinate errors, timing errors and the bias

error between the tracker and DME local survey positions. With the exception of the last error, which is assumed to be composed of systematic biases(δX, δY, δZ), the errors are assumed to be normally distributed about a zero mean (random). The character of the errors and adopted symbolization is presented in Table II-1.

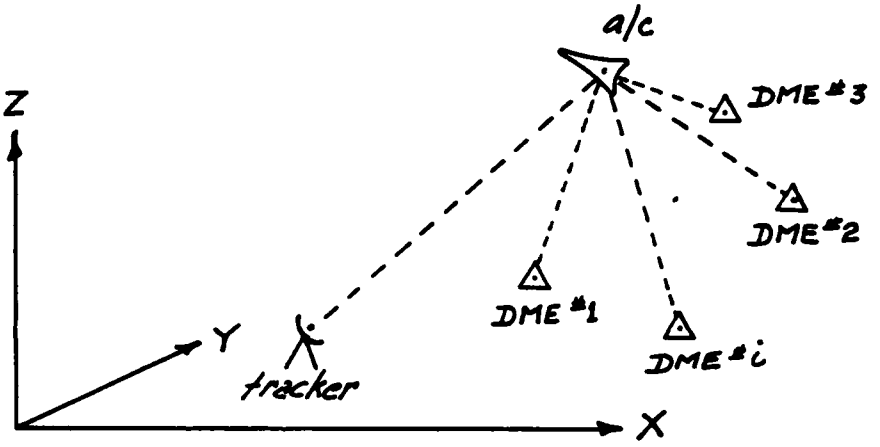


Figure II-1 Trilateration Network

ERROR SOURCE	RANDOM	SYSTEMATIC
Range by DME	σ_r	$\delta r \equiv 0$
Survey coord. of DME units	$\sigma_{X_i}, \sigma_{Y_i}, \sigma_{Z_i}$	$\delta X_i \equiv \delta Y_i \equiv \delta Z_i \equiv 0$
Timing	σ_t	$\delta t \equiv 0$
Survey coord. of tracker w/r DME	$\sigma_{X_t} \equiv \sigma_{Y_t} \equiv \sigma_{Z_t} \equiv 0$	δX_t , δY_t , δZ_t

TABLE II-1 Error Sources and Symbolization

B. MATHEMATICAL MODELS

The models required for the error analysis relate the observed quantities as the dependent variables to the unknown and given quantities. Working in three-dimensional rectangular coordinates, the observations of range can be related to the position of the a/c at any time (t) from any DME station (i) by:

$$f(r_{it}) = r_{it_{obs}} - \left\{ [X_i - X_t]^2 + [Y_i - Y_t]^2 + [Z_i - Z_t]^2 \right\}^{1/2} = 0 \qquad (2\text{-}1)$$

However, all the DME units do not measure range simultaneously. Consequently, it is necessary to express the spatial track of the a/c for a short period in terms of a line segment in space. The symmetric form of a line in cartesian space is:

$$\frac{X_1 - X_2}{a} = \frac{Y_1 - Y_2}{b} = \frac{Z_1 - Z_2}{c} \qquad (2\text{-}2)$$

where: (1) and (2) represent two points on a line segment.

We now assign (X_o, Y_o, Z_o) as coordinates of the a/c at about the midpoint of the line segment track. At any other time, the a/c coordinates (X_t, Y_t, Z_t) can be represented as:

$$X_t = X_o + \dot{X}\ \Delta t \qquad (2\text{-}3\ a)$$

$$Y_t = Y_o + \dot{Y}\ \Delta t \qquad (2\text{-}3\ b)$$

$$Z_t = Z_o + \dot{Z}\ \Delta t \qquad (2\text{-}3\ c)$$

where:

$\dot{X}, \dot{Y}, \dot{Z}$ represent the time rate of change of a/c along the respective axes

$\Delta t \equiv$ time at (t) - t_o

$t_o \equiv$ the time when the a/c is at (X_o, Y_o, Z_o)

In equation (2-2), the points can be replaced by the origin point (X_o, Y_o, Z_o) and any other point at time (t). ie (X_t, Y_t, Z_t).

i.e.:

$$\frac{X_o - (X_o + \dot{X}\Delta t)}{a} = \frac{Y_o - (Y_o + \dot{Y}\Delta t)}{b} = \frac{Z_o - (Z_o + \dot{Z}\Delta t)}{c} \qquad (2\text{-}4)$$

After some derivation there results:

$$X_t = X_o + \frac{a}{b}(\dot{Y}\ \Delta t) \qquad (2\text{-}5a)$$

$$Y_t = Y_o + \frac{b}{c}(\dot{Z}\ \Delta t) \qquad (2\text{-}5b)$$

$$Z_t = Z_o + \frac{c}{a}(\dot{X}\ \Delta t) \qquad (2\text{-}5c)$$

Substituting this result into equation (2-1):

$$f(r_{it}) = r_{it} - \left\{ \left[X_i - X_o - \frac{a}{b}(\dot{Y}\Delta t) \right]^2 + \left[Y_i - Y_o - \frac{b}{c}(\dot{Z}\Delta t) \right]^2 + \left[Z_i - Z_o - \frac{c}{a}(\dot{X}\Delta t) \right]^2 \right\}^{1/2} = 0 \quad (2\text{-}6)$$

Equation (2-6) represents the mathematical model relating observations of range by DME (i) at time (t) to the a/c positional parameters of the trilateration network.

To facilitate the introduction of error estimates of the DME station coordinates in a local survey system, it is convenient to treat the survey coordinates as if they were directly observed quantities. Their weights can then be used in the adjustment computation to introduce their contribution to the system error. In this case of direct observations of unknown parameters, the mathematical models are simply:

$$f(X_i) = X_{i_{obs}} - X_{i_{adj}} = 0 \quad (2\text{-}7a)$$

$$f(Y_i) = Y_{i_{obs}} - Y_{i_{adj}} = 0 \quad (2\text{-}7b)$$

$$f(Z_i) = Z_{i_{obs}} - Z_{i_{adj}} = 0 \quad (2\text{-}7c)$$

where:

"obs" pertains to the observed value

"adj" pertains to the adjusted value

The mathematical models which relate the observations of DME range and local survey coordinates to the parameters defining a/c position are the Equations (2-6) and (2-7).

C. OBSERVATION EQUATIONS

The vehicle through which this analysis of errors is conducted is the adjustment computation using least squares. For this, it is necessary to arrange the mathematical models in linear form with respect to the unknown parameters. For this, the model is represented by a Taylor Series expansion neglecting second degree and higher terms. A residual is appended to each observation to account for the specific observational error. These residuals are assumed to be normally distributed about a zero mean, a condition which is reasonably well-attained provided the mathematical models represent the system. In this case, after the series expansion of the models, the observation equations are: [Merchant, 1974]

$$V + B\Delta + E = 0 \qquad (2\text{-}8)$$

After a logical grouping of parameters, the observation equations pertaining to observations of DME range are:

$$\overset{r}{V} + \overset{s}{B}\overset{s}{\Delta} + \overset{p}{B}\overset{p}{\Delta} + \overset{f}{B}\overset{f}{\Delta} + \overset{r}{E} = 0 \qquad (2\text{-}9)$$

where: (see Appendix B)

$\overset{r}{V}$ ≡ vector of obs. residuals of DME range.

$\overset{s}{B} \equiv \left[\partial f(r_{it})/\partial(X_i, Y_i, Z_i)\right]$ for all (i)

$\overset{p}{B} \equiv \left[\partial f(r_{it})/\partial(X_o, Y_o, Z_o, \dot{X}_o, \dot{Y}_o, \dot{Z}_o)\right]$

$\overset{c}{B} \equiv \left[\partial f(r_{it})/\partial(a, b, c)\right]$

$\overset{s'}{\Delta} \equiv \left[\delta X_i, \delta Y_i, \delta Z_i\right]$ for all (i)

$\overset{p'}{\Delta} \equiv \left[\delta X_o, \delta Y_o, \delta Z_o, \delta \dot{X}_o, \delta \dot{Y}_o, \delta \dot{Z}_o\right]$

$\overset{c'}{\Delta} \equiv \left[\delta a, \delta b, \delta c\right]$

$\overset{c}{E}$ = A vector of f (r_{it}) evaluated for station (i) at time (t) using the observed DME range and current estimates of all parameters in Equation (2-6).

The observation equations pertaining to the direct observations on the local survey coordinates of DME stations are: [Merchant, 1974]

$$\overset{s}{V} - \overset{s}{\Delta} + \overset{s}{E} = 0 \qquad (2\text{-}10)$$

where:

$\overset{s}{V}$ ≡ vector of obs. residuals on direct survey coordinate observations (X_i, Y_i, Z_i) for all stations (i).

$\overset{s}{E}$ ≡ a vector of f $(X, Y, Z)_i$ evaluated for station (i) using original obs. and current estimates of all parameters. [equations (2-7)]

Collecting the observation equations (2-9) and (2-10), they are expressed as:

$$\overline{V} + \overline{B}\,\overline{\Delta} + \overline{E} = 0 \qquad (2\text{-}11)$$

where:

$$\bar{B} = \begin{bmatrix} {}^{s}\bar{B} & {}^{p}\bar{B} & {}^{f}\bar{B} \\ -I & 0 & 0 \end{bmatrix} \quad ; \qquad \bar{\Delta} = \begin{bmatrix} {}^{s}\Delta \\ {}^{p}\Delta \\ {}^{f}\Delta \end{bmatrix}$$

$$\bar{V} = \begin{bmatrix} {}^{r}V \\ {}^{s}V \end{bmatrix} \quad ; \qquad \bar{E} = \begin{bmatrix} {}^{r}E \\ {}^{s}E \end{bmatrix}$$

D. NORMAL EQUATIONS

The number of observation equations normally exceeds the number of unknown parameters. As a consequence, Equations (2-11) have no unique solution. By imposing on these equations the condition that the sum of the weighted squared residuals be a minimum ("least squares"), a solution is uniquely possible provided the unknown parameters are linearly independent. This condition on Equations (2-11) leads to the "normal equations":

$$\bar{N}\bar{\Delta} + \bar{U} = 0 \qquad (2\text{-}12)$$

However, since we are interested now only in an analysis of errors as they propagate through to the results, we need not solve for the solution vector (Δ). We need only form and invert ($\bar{N}$) since:

$$\bar{N}^{-1} = \Sigma$$

where:

$\Sigma \equiv$ the estimate of variance/covariance of the adjusted parameters.

Accordingly, we are interested only in the formation and inverse of $(\bar{N})$, the normal coefficient matrix. It can be shown that: [Merchant, 1974]

$$\bar{N} = \bar{B}' \bar{W} \bar{B} \qquad (2\text{-}13)$$

The normal coefficient matrix $(\bar{N})$ can be simplified by exploiting the sparse patterns of $(\bar{B})$ and $(\bar{W})$. The result:

$$\bar{N} = \begin{bmatrix} \overset{s}{B}'\overset{r}{W}\overset{s}{B} + \overset{s}{W} & \overset{s}{B}'\overset{r}{W}\overset{p}{B} & \overset{s}{B}'\overset{r}{W}\overset{f}{B} \\ \overset{p}{B}'\overset{p}{W}\overset{p}{B} & \overset{p}{B}'\overset{r}{W}\overset{f}{B} & \\ \overset{f}{B}'\overset{r}{W}\overset{f}{B} & & \end{bmatrix} \qquad (2\text{-}14)$$

(See Appendix A for derivations of elements of $\bar{N}$)

Provided $|\bar{N}| \neq 0$, a unique solution for the system of 18 equations exists. The computation of the inverse of the symmetrical matrix $(\bar{N})$ is a modest burden. Accordingly, no further attempts are made to further exploit the sparse patterns that exist in $(\bar{N})$ for purposes of computing $(\bar{N}^{-1})$.

E. RANDOM POSITIONAL ERRORS

By extracting the array $(\overset{p}{\Sigma})$ within (Σ) related to a/c position (ie $\dot{X}_o$, $\dot{Y}_o$, $\dot{Z}_o$, X_o, Y_o, Z_o), the error in a/c position at any time (t) can be determined by propagation of $(\overset{p}{\Sigma})$ through the following functions:

$$X_t = X_0 + \dot{X}_0 \, \Delta t \qquad (2\text{-}15a)$$

$$Y_t = Y_0 + \dot{Y}_0 \, \Delta t \qquad (2\text{-}15b)$$

$$Z_t = Z_0 + \dot{Z}_0 \, \Delta t \qquad (2\text{-}15c)$$

F. RANDOM TIMING ERRORS

Errors are assumed for the time of observation (Δt) as well as for the positional parameters. Therefore, the variance/covariance matrix will be:

$$\overset{\$}{\Sigma} = \begin{bmatrix} \overset{P}{\Sigma} & 0 \\ 0 & \sigma^2_{t_a} \end{bmatrix}_{7\times 7} \qquad (2\text{-}16)$$

G. VARIANCE/COVARIANCE OF A/C POSITION

Finally, the estimated variance/covariance for a/c position at any time (t) due to errors in DME observation, time observation and survey (local) coordinates can be computed by propagation through the related function.

$$\overset{*}{\Sigma} = G \overset{\$}{\Sigma} G' \qquad (2\text{-}17)$$

where:

$$G = \begin{bmatrix} \partial X_t/\partial(X_o Y_o Z_o \dot{X}_o \dot{Y}_o \dot{Z}_o \Delta t) \\ \partial Y_t/\partial(X_o Y_o Z_o \dot{X}_o \dot{Y}_o \dot{Z}_o \Delta t) \\ \partial Z_t/\partial(X_o Y_o Z_o \dot{X}_o \dot{Y}_o \dot{Z}_o \Delta t) \end{bmatrix}_{3\times 7}$$

$$\overset{*}{\Sigma} = \begin{bmatrix} \underline{\sigma^2_{X_t}} & \sigma_{X_t Y_t} & \sigma_{X_t Z_t} \\ & \underline{\sigma^2_{Y_t}} & \sigma_{Y_t Z_t} \\ & & \sigma^2_{Z_t} \end{bmatrix}_{3\times 3}$$

ie: The variance/covariance matrix for the spatial coordinates of the a/c at the sample point time (t) for error sources of:

- DME range
- DME spatial coordinates
- time

H. RESULTING RANDOM AND SYSTEMATIC POSITIONAL ERRORS

The foregoing discussion concerned the random components of error producing sources in the trilateration procedure. The primary systematic error sources in the trilateration procedure are the fixed or bias errors along coordinate axes relating the local survey of the DME gound stations to the position of the "tracker". These error values depend on the means by which they are determined. These vary with each application. Since the component bias errors of the tracker to DME distances cannot be combined with the remaining random components of the trilateration procedure, the

total system error for a/c position at any time can be expressed as:

$$X_t \text{ error} = \sigma X_t + \delta X_t \qquad (2\text{-}18a)$$

$$Y_t \text{ error} = \sigma Y_t + \delta Y_t \qquad (2\text{-}18b)$$

$$Z_t \text{ error} = \sigma Z_t + \delta Z_t \qquad (2\text{-}18c)$$

III. CONCLUSION

The purpose of this report has been to present the concept, theory and mathematical basis for assessing the combined errors of a DME ground based tracking system on the relative position of a "target" aircraft with respect to a ground based tracker.

The component errors of the system that have been included are those of error in DME passive range observations, local geodetic survey coordinates of the ground based DME units and system timing errors. The result is in terms of an algebraic expression of the variance/covariance of the aircraft with respect to the tracker's position. In addition, provision was made for the introduction of the geodetic bias error which exists between the tracker and the local DME ground survey.

Subsequent work to meet the overall objective should include, in order:

- analysis of the design matrix ($\bar{B}$) to assure linear independence
- programming of the Equations (2-14, and 2-17)
- generating plots of positional errors exercising the geometry of the network and the magnitude of contributing error values which are representative of anticipated applications.

APPENDIX A

ELEMENTS OF THE DESIGN AND WEIGHT MATRICES

$\overset{s}{B}$ - for local DME survey coordinates

- further breakdown:

$$\overset{s}{B} = \begin{bmatrix} \overset{s}{B}_1 & & & 0 \\ & \overset{s}{B}_a & & \\ & & \overset{s}{B}_i & \\ 0 & & & \overset{s}{B}_m \end{bmatrix}$$

where # DME sta i= 1 to n

for $n \geq 3$

$$\overset{s}{B}_i = \left[\partial f(r_{ij}) / \partial(X_i Y_i Z_i)\right]_{j \times 3}$$

where j = 1 to m_i

for m_i DME obs. from

sta. (i).

$$\overset{s}{B}_i(j,1) = (\partial f(r_{ij})/\partial X_i)$$
$$\overset{s}{B}_i(j,2) = (\partial f(r_{ij})/\partial Y_i)$$
$$\overset{s}{B}_i(j,3) = (\partial f(r_{ij})/\partial Z_i)$$

$\overset{p}{B}$ - for parameters of a/c position

- further breakdown:

$$\overset{p}{\mathcal{B}} = \begin{bmatrix} \overset{p}{B}_1 \\ \overset{p}{B}_a \\ \vdots \\ \overset{p}{B}_i \\ \vdots \\ B_n \end{bmatrix}$$

$$\overset{p}{B}_i = \underset{j \times 6}{\left[\partial f(r_{ij}) / \partial (X_o, Y_o, Z_o, \dot{X}_o, \dot{Y}_o, \dot{Z}_o)\right]}$$

ie: $\overset{p}{B}_i(j,1) = (\partial f(r_{ij}) / \partial X_o)$

$\equiv$

$\overset{p}{B}_i(j,6) = (\partial f(r_{ij}) / \partial \dot{Z}_o)$

$\overset{f}{B}$
B - for parameters describing spatial line segment

- further breakdown:

$$\overset{f}{B} = \begin{bmatrix} \overset{f}{B}_1 \\ \overset{f}{B}_a \\ \vdots \\ \overset{f}{B}_i \\ \vdots \\ \overset{f}{B}_n \end{bmatrix}$$

$$\overset{f}{B}_i = \left[\partial f(r_{ij}) / \partial(a,b,c) \right]_{j \times 3}$$

ie: $\overset{f}{B}_i(j,1) = \left[\partial f(r_{ij}) / \partial a \right]$

$\overset{f}{B}_i(j,2) = \left[\partial f(r_{ij}) / \partial b \right]$

$\overset{f}{B}_i(j,3) = \left[\partial f(r_{ij}) / \partial c \right]$

$\overset{r}{W}$ - Weight matrix for DME range observations:

Ideally $$\overset{r}{W} = \overset{r}{\Sigma}^{-1}$$

However, since correlations between observations of range should be low, provided careful modeling has been accomplished, the weight matrix can be taken as a diagonal matrix whose elements are the inverse of the standard error (standard deviation), squared in the units in which the observation was modeled. That is:

$$\overset{r}{W} = \overset{r}{\Sigma}^{-1} \longrightarrow \begin{bmatrix} \overset{r}{W}_1 & & & 0 \\ & \overset{r}{W}_2 & & \\ & & \overset{r}{W}_i & \\ 0 & & & \overset{r}{W}_n \end{bmatrix}$$

where:

$$\overset{r}{W}_i = \begin{bmatrix} \frac{1}{\sigma^2_{r_1}} & & & 0 \\ & \frac{1}{\sigma^2_{r_j}} & & \\ & & \frac{1}{\sigma^2_{rm_i}} & \\ 0 & & & \end{bmatrix}$$

$\overset{s}{W}$ - weight matrix for local survey coordinates of the DME ground station locations

Ideally, $\overset{s}{W} = \overset{s}{\Sigma}^{-1}$; i.e., the inverse of the full variance/covariance matrix which relate all station coordinates. However, a valid estimate of ($\overset{s}{\Sigma}$) is rarely available. Consequently, the correlation between coordinate values of survey points is normally neglected. Accordingly;

$$\overset{s}{W} = \begin{bmatrix} \frac{1}{\sigma^2_{x_1}} & & & & & & 0 \\ & \frac{1}{\sigma^2_{y_1}} & & & & & \\ & & \frac{1}{\sigma^2_{z_1}} & & & & \\ & & & \frac{1}{\sigma^2_{x_2}} & & & \\ & & & & \ddots & & \\ & & & & & \frac{1}{\sigma^2_{x_i}} & \\ 0 & & & & & & \ddots \\ & & & & & & & \frac{1}{\sigma^2_{z_n}} \end{bmatrix}$$

APPENDIX B

DEFINITIONS OF TERMS

a/c	≡	aircraft
σ	≡	standard deviation of normally distributed quantity about a zero mean
σ^2	≡	variance
$\sigma_{x,y}$	≡	covariance of x with y
δ	≡	systematic bias error
X, Y, Z	≡	coordinates in a three dimensional, rectangular, right handed system
t	≡	range time
a, b, c	≡	constants appearing in the symmetric form of an equation expressing line in space
X_t, Y_t, Z_t	≡	position of a/c at time (t)
X_o, Y_o, Z_o	≡	position of a/c at time (t_o)
Σ	≡	variance/covariance matrix
X_i, Y_i, Z_i	≡	coordinates of DME station (i)

APPENDIX C

Reference

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Dept. of Geodetic Science, The Ohio State University, June 1974.

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